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On the way to Brane New World[1]

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ABSTRACT

In this report we consider brane-world universe (New Brane World) where an arbitrary large N quantum CFT exists on the domain wall. This corresponds to implementing of Randall-Sundrum compactification within the context of AdS/CFT correspondence. Using anomaly induced effective action for domain wall CFT, the possibility of self-consistent quantum creation of 4d de Sitter wall universe (inflation) is demonstrated. In case of maximally SUSY Yang-Mills theory the exact correspondence with radius and effective tension found by Hawking-Hertog-Reall is obtained.

We also discuss the bosonic sector of 5d gauged supergravity with single scalar and taking the boundary action as predicted by supersymmetry and discuss the possibility to supersymmetrize dilatonic New Brane World. It is demonstrated that for a number of superpotentials the flat SUSY dilatonic brane-world (with dynamically induced brane dilaton) or quantum-induced de Sitter dilatonic brane-world (not Anti-de Sitter one) where SUSY is broken by the quantum effects occurs. The analysis of graviton perturbations indicates that gravity is localized on such branes.

New Brane World is useful in the study of FRW dynamics and cosmological entropy bounds. Brane stress tensor is induced by quantum effects of dual CFT and brane crosses the horizon of AdS black hole. The similarity between CFT entropy at the horizon and FRW equations is extended on the quantum level. This suggests the way to understand cosmological entropy bounds in quantum gravity.

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1 Introduction

The word "Brane New World" is the title of the paper by Hawking-Hertog-Reall [1]. After the discovery that gravity on the brane may be localized [2], there was renewed interest in the studies of higher-dimensional (brane-world) theories. In particular, numerous works [3] (and refs. therein) have been devoted to the investigation of cosmology (inflation) of brane-worlds. In refs.[1, 4, 5] it has been suggested that the inflationary brane-world scenario could be realized due to quantum effects of brane matter. Such a scenario is based on the large N quantum CFT living on the brane [1, 4, 5]. Unlike to convenient brane-worlds, the boundary action is not the arbitrary one (the brane tension is not a free parameter). On the contrary, the surface terms on AdS-like space are motivated by the AdS/CFT correspondence. Their role is in making of variational procedure to be well-defined and in the elimination of the leading divergence of the AdS-like action. In accordance with AdS/CFT correspondence, there is quantum CFT living on the brane. Such brane quantum CFT produces conformal anomaly which leads to creation of effective brane tension. As a result, the dynamical mechanism to get flat or curved (de Sitter or Anti-de Sitter) brane-world appears [1, 4, 5] in frames of AdS/CFT duality. Hence, one gets less fine-tuning in realization of brane-worlds as brane tension is not free parameter. The nice feature of this dynamical scenario is that the sign of conformal anomaly terms for usual matter predicts de Sitter (inflationary) universe as a preferrable solution in one-brane case.

From another side, there is much activity now in the supersymmetrization of Randall-Sundrum brane world [6, 7, 8, 9] (see also refs. therein). The 5d gauged supergravity represents very interesting model where supersymmetric dilatonic brane-world should be searched. Moreover, in such model, it is natural to try to construct supersymmetric dilatonic brane-world consistent with AdS/CFT correspondence [10]. It could be then that such a scenario should be realized as supersymmetric version of New Brane World [1, 4, 5, 11].

Brane New World may find further applications. Indeed, it is quite well-known fact that holographic principle suggests the interesting bounds between microscopic and Bekenstein-Hawking entropy [12] as it was discussed in refs.[13, 14]. Recently, the very interesting attempt to study the holographic principle in Friedmann-Robertson-Walker (FRW) universe filled by CFT has been done by Verlinde [15]. Using dual AdS-description [10] it has been found the relation between the entropy (energy) of CFT and the cosmological equations of motion in FRW universe. In particulary, the equation controlling the entropy bounds during evolution has been obtained [15] and Cardy-Verlinde formula has been derived. These results have been subsequently generalized and discussed in a number of works [16, 17].

One interesting extension has been presented in ref.[18] where similar questions have been studied from classical brane-world perspective[2, 19]. In particulary, the behaviour of the CFT entropy at the horizon of bulk 5d AdS BH has been investigated and its comparison with FRW equations has been done. In the present report, based on [20], we also generalize the situation described in ref.[18] to the case of quantum-induced (or AdS/CFT induced) brane-worlds suggested in refs.[1, 4, 5]. In this way, from one side, one gets quantum-corrected FRW universe equations as they look from the point of view of not only brane observer (who knows nothing about bulk 5d BH) but also from the point of view of quantum induced brane-world. From another side, one gets the quantum-corrected brane entropy as well as Hubble constant and Hawking temperature at the horizon. Finally, this may be considered as extension of scenario of refs.[1, 4, 5] (see refs.[21] for related questions) which admits also generalization for the presence of non-trivial dilaton and (or) supersymmetrization [11] for the case when brane crosses the horizon of AdS-black hole.

This report is organized as follows. In the next section, the equivalence between 5d dilatonic gravity and 4d dilatonic gravity coupled with CFT is discussed. In section 3, based on [5], we give the inflationary brane-world scenario realized due to quantum effects of brane matter by using anomaly induced effective action. As a result, one can consider the arbitrary content of CFT living on the wall. Moreover, the formalism is applied not only to 4d de Sitter wall but also to 4d hyperbolic wall in 5d Anti-de Sitter universe or 4d conformally flat universe. In section 4, the review of the construction of classical supersymmetric brane-world is done for bosonic sector of 5d gauged SG with single dilaton. Boundary action is predicted by supersymmetry. Half of supersymmetries survives for flat brane-world (as it follows from the analysis of BPS condition). The classical SUSY curved brane-worlds cannot be realized. Fifth section is devoted to the extension of the analysis of fourth section modified by the quantum contribution from brane CFT in order to construct SUSY New Brane-World. It is shown for number of superpotentials that, unlike to classical case, the quantum induced de Sitter brane-world is created. However, the brane supersymmetry is broken by quantum effects. The example of SUSY flat brane-world, where boundary value of dilaton is defined by quantum effects, is also given. In section 6, the analysis of graviton perturbations around found solutions is done. It is shown that only one normalizable solution corresponding to zero mode exists. In other words, gravity should be trapped on the brane in such scenario. In section 7, we consider the generalization of approach of ref.[18] to the case of quantum-induced brane-worlds [1, 4, 5] and obtain a quantum-corrected FRW universe equations. Quantum-corrected Hubble constant, Hawking temperature and cosmological entropy are found on the FRW brane. Some brief summary and outlook is given in final section.

2 AdS/CFT and the localization of the gravity

$\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence tells us that the effective action $W_{\rm CFT}$ of CFT in 4 dimensions is given by the path integral of the supergravity in 5 dimensional AdS space:

$$\begin{aligned}
&\mathrm{e}^{-W_{\rm CFT}}=\int[dg][d\varphi]\mathrm{e}^{-S_{\rm grav}}\ ,\quad S_{\rm grav}=S_{\rm EH}+S_{\rm GH}+S_1+S_2+\cdots\ , \\
&S_{\rm EH}=\frac{1}{16\pi G}\int d^5x\sqrt{g_{(5)}}\left(R_{(5)}+\frac{12}{l^2}+\cdots\right)\ ,\quad S_{\rm GH}=\frac{1}{8\pi G}\int d^4x\sqrt{g_{(4)}}\nabla_\mu n^\mu\ , \\
&S_1=-\frac{1}{8\pi G}\int d^4x\sqrt{g_{(4)}}\left(\frac{3}{l}+\cdots\right)\ ,\quad S_2=-\frac{l}{16\pi G}\int d^4x\sqrt{g_{(4)}}\left(\frac{1}{2}R_{(4)}+\cdots\right)\ .
\end{aligned}\tag{1}$$

Here φ expresses the (matter) fields besides the graviton. $S_{\rm EH}$ corresponds to the Einstein-Hilbert action and $S_{\rm GH}$ to the Gibbons-Hawking surface counter term and n^μ is the unit vector normal to the boundary. S_1, S_2, $\cdots$ correspond to the surface counter terms, which cancell the divergences when the boundary in AdS_5 goes to the infinity.[4]

In [1], two 5 dimensional balls $\mathrm{B}_5^{(1,2)}$ are glued on the boundary, which is 4 dimensional sphere S_4[5]. Instead of $S_{\rm grav}$, if one considers the following action S

$$S = S_{\rm EH} + S_{\rm GH} + S_1 = S_{\rm grav} - S_2 - \cdots , \tag{2}$$

for two balls, using (1), one gets the following boundary theory in terms of the partition function [1]:

$$\begin{aligned} &\int_{\mathrm{B}_5^{(1)}+\mathrm{B}_5^{(1)}+\mathrm{S}_4}[dg][d\varphi]\mathrm{e}^{-S} = \left(\int_{\mathrm{B}_5}[dg][d\varphi]\mathrm{e}^{-S_{\rm EH}-S_{\rm GH}-S_1}\right)^2 \\ &= \mathrm{e}^{2S_2+\cdots}\left(\int_{\mathrm{B}_5}[dg][d\varphi]\mathrm{e}^{-S_{\rm grav}}\right)^2 = \mathrm{e}^{-2W_{\rm CFT}+2S_2+\cdots} . \end{aligned} \tag{3}$$

Since S_2 can be regarded as the Einstein-Hilbert action on the boundary, which is S_4 in the present case, the gravity on the boundary becomes dynamical. The 4 dimensional gravity is nothing but the gravity localized on the brane in the Randall-Sundrum model [2].

For $\mathcal{N}=4$ $SU(N)$ Yang-Mills theory, the AdS/CFT dual is given by identifying

$$l = g_{\rm YM}^{\frac{1}{2}} N^{\frac{1}{4}} l_s , \quad \frac{l^3}{G} = \frac{2N^2}{\pi} . \tag{4}$$

Here $g_{\rm YM}$ is the coupling of the Yang-Mills theory and l_s is the string length. Then (3) tells that the RS model is equivalent to a CFT ($\mathcal{N}=4$ $SU(N)$ Yang-Mills theory) coupled to 4 dimensional gravity including some correction coming from the higher order counter terms with a Newton constant given by $G_4 = G/l$. This is an excellent explanation [1] to why gravity is trapped on the brane.

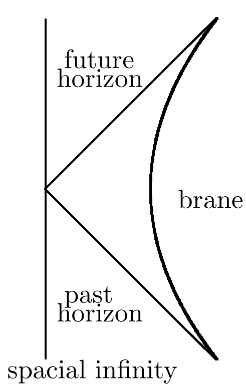
3 Brane New World

In [2], the discussion was limited by the flat brane. In this case, however, the brane crosses the event horizon in the finite time, which opens the causality problem[6]. To avoid

[4]See [23] for the surface counterterms in dilaton coupled supergravities.

[5]The reason why this situation was considered is given in the next section. In this section, we can consider the case that the brane is the boundary of two AdS spaces.

[6]We have the following causal structure.



this problem, it would be natural to consider de Sitter brane which is also motivated by cosmology[1]. If the brane is de Sitter space, the brane does not cross the horizon. Motivated by this, we consider the curved brane in this section.

Let us take the spacetime whose boundary is 4 dimensional sphere S_4, which can be identified with a D3-brane. The bulk part is given by 5 dimensional Euclidean anti de Sitter space AdS_5[7]

$$ds^2_{AdS_5} = dy^2 + l^2 \sinh^2 \frac{y}{l} d\Omega_4^2 \ . \tag{5}$$

Here $d\Omega_4^2$ is given by the metric of S_4 with unit radius. One also assumes the boundary (brane) lies at $y = y_0$ and the bulk space is given by gluing two regions given by $0 \leq y < y_0$.

We start with the action S which is the sum of the Einstein-Hilbert action S_{EH}, the Gibbons-Hawking surface term S_{GH}, the surface counter term S_1 and the trace anomaly induced action $\mathcal{W}$[8]:

$$\begin{aligned}
& S = S_{EH} + S_{GH} + 2S_1 + \mathcal{W} \ , \quad S_{EH} = \frac{1}{16\pi G}\int d^5x\sqrt{g_{(5)}}\left(R_{(5)} + \frac{12}{l^2}\right) \ , \\
& S_{GH} = \frac{1}{8\pi G}\int d^4x\sqrt{g_{(4)}}\nabla_\mu n^\mu \ , \quad S_1 = -\frac{3}{8\pi G l}\int d^4x\sqrt{g_{(4)}} \ , \\
& \mathcal{W} = b\int d^4x\sqrt{\widetilde{g}}\widetilde{F}A + b'\int d^4x\sqrt{\widetilde{g}}\left\{A\left[2\widetilde{\Box}^2 + \widetilde{R}_{\mu\nu}\widetilde{\nabla}_\mu\widetilde{\nabla}_\nu - \frac{4}{3}\widetilde{R}\widetilde{\Box}^2 \right.\right. \\
& \qquad \left.\left. + \frac{2}{3}(\widetilde{\nabla}^\mu\widetilde{R})\widetilde{\nabla}_\mu\right]A + \left(\widetilde{G} - \frac{2}{3}\widetilde{\Box}\widetilde{R}\right)A\right\} \\
& \qquad -\frac{1}{12}\left\{b'' + \frac{2}{3}(b + b')\right\}\int d^4x\sqrt{\widetilde{g}}\left[\widetilde{R} - 6\widetilde{\Box}A - 6(\widetilde{\nabla}_\mu A)(\widetilde{\nabla}^\mu A)\right]^2 \ .
\end{aligned} \tag{6}$$

Here the quantities in the 5 dimensional bulk spacetime are specified by the suffices $_{(5)}$ and those in the boundary 4 dimensional spacetime are by $_{(4)}$. The factor 2 in front of S_1 in (6) is coming from that we have two bulk regions which are connected with each other by the brane. In (6), n^μ is the unit vector normal to the boundary. In (17), one chooses the 4 dimensional boundary metric as

$$g_{(4)\mu\nu} = e^{2A}\tilde{g}_{\mu\nu} \tag{7}$$

and we specify the quantities with $\tilde{g}_{\mu\nu}$ by using $\tilde{\ }$. G ($\tilde{G}$) and F ($\tilde{F}$) are the Gauss-Bonnet invariant and the square of the Weyl tensor, which are given as [9]

$$G = R^2 - 4R_{ij}R^{ij} + R_{ijkl}R^{ijkl} \ , \quad F = \frac{1}{3}R^2 - 2R_{ij}R^{ij} + R_{ijkl}R^{ijkl} \ , \tag{8}$$

[7]For the expressions of the metric of AdS, see Appendix.

[8]For the introduction to anomaly induced effective action in curved space-time (with torsion), see section 5.5 in [24].

[9]We use the following curvature conventions:

$$\begin{aligned}
& R = g^{\mu\nu}R_{\mu\nu} \ , \quad R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu} \\
& R^\lambda{}_{\mu\rho\nu} = -\Gamma^\lambda_{\mu\rho,\nu} + \Gamma^\lambda_{\mu\nu,\rho} - \Gamma^\eta_{\mu\rho}\Gamma^\lambda_{\nu\eta} + \Gamma^\eta_{\mu\nu}\Gamma^\lambda_{\rho\eta} \ , \quad \Gamma^\eta_{\mu\lambda} = \frac{1}{2}g^{\eta\nu}\left(g_{\mu\nu,\lambda} + g_{\lambda\nu,\mu} - g_{\mu\lambda,\nu}\right) \ .
\end{aligned}$$

In the effective action (17) induced by brane quantum matter, in general, with N real scalar, $N_{1/2}$ Dirac spinor, N_1 vector fields, N_2 (= 0 or 1) gravitons and $N_{\rm HD}$ higher derivative conformal scalars, b, b' and b'' are[10]

$$\begin{aligned} b &= \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{\rm HD}}{120(4\pi)^2} , \\ b' &= -\frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{\rm HD}}{360(4\pi)^2} , \quad b'' = 0 . \end{aligned} \tag{9}$$

Usually, b'' may be changed by the finite renormalization of local counterterm in the gravitational effective action. As it was the case in ref.[11], the term proportional to $\left\{b'' + \frac{2}{3}(b+b')\right\}$ in (17), and therefore b'' does not contribute to the equations of motion.

For typical examples motivated by AdS/CFT correspondence[10] one has: a) $\mathcal{N}=4$ $SU(N)$ SYM theory[11] $b=-b'=\frac{C}{4}=\frac{N^2-1}{4(4\pi)^2}$, b) $\mathcal{N}=2$ $Sp(N)$ theory[12] $b=\frac{12N^2+18N-2}{24(4\pi)^2}$, $b'=-\frac{12N^2+12N-1}{24(4\pi)^2}$. Note that b' is negative in the above cases.

We should also note that $\mathcal{W}$ in (17) is defined up to conformally invariant functional, which cannot be determined from only the conformal anomaly. The conformally flat space is a pleasant exclusion where anomaly induced effective action is defined uniquely. However, one can argue that such conformally invariant functional gives next to leading contribution as mass parameter of regularization may be adjusted to be arbitrary small (or large).

The metric of S_4 with the unit radius is given by

$$d\Omega_4^2 = d\chi^2 + \sin^2\chi d\Omega_3^2 . \tag{10}$$

Here $d\Omega_3^2$ is described by the metric of 3 dimensional unit sphere. If we change the coordinate χ to σ by $\sin\chi = \pm\frac{1}{\cosh\sigma}$, one obtains[13]

$$d\Omega_4^2 = \frac{1}{\cosh^2\sigma}\left(d\sigma^2 + d\Omega_3^2\right) . \tag{11}$$

[10] These parameters appear in the general expression of the conformal anomaly T

$$T = b\left(F + \frac{2}{3}\Box R\right) + b'G + b''\Box R .$$

[11] A multiplet of $\mathcal{N}=4$ theory contains 1 vector, 4 Majorana spinors (2 Dirac spinors) and 6 real scalars.

[12] $\mathcal{N}=2$ $Sp(N)$ theory contains $2N^2+N$ vector multiplets and $2N^2+7N-1$ hypermultiplets. One vector multiplet contains 1 vector, 2 Majorana spinors (1 Dirac spinor) and 2 real scalars. On the other hand, one hypermultiplet does not contain a vector but 2 Majorana spinors (1 Dirac spinor) and 4 real (2 complex) scalars.

[13] If we Wick-rotate the metric by $\sigma \to it$, we obtain the metric of de Sitter space:

$$d\Omega_4^2 \to ds_{\rm dS}^2 = \frac{1}{\cos^2 t}\left(-dt^2 + d\Omega_3^2\right) .$$

On the other hand, the metric of the 4 dimensional flat Euclidean space is given by

$$ds^2_{\rm 4E} = d\rho^2 + \rho^2 d\Omega_3^2 \ . \tag{12}$$

Then by changing the coordinate as $\rho = {\rm e}^\sigma$, one gets

$$ds^2_{\rm 4E} = {\rm e}^{2\sigma}\left(d\sigma^2 + d\Omega_3^2\right) \ . \tag{13}$$

For the 4 dimensional hyperboloid with the unit radius, the metric is given by

$$ds^2_{\rm H4} = d\chi^2 + \sinh^2\chi d\Omega_3^2 \ . \tag{14}$$

Changing the coordinate χ to σ by $\sinh\chi = \frac{1}{\sinh\sigma}$, one finds

$$ds^2_{\rm H4} = \frac{1}{\sinh^2\sigma}\left(d\sigma^2 + d\Omega_3^2\right) \ . \tag{15}$$

Let us now discuss the 4 dimensional hyperboloid whose boundary is the 3 dimensional sphere S_3 but we can consider the cases that the boundary is a 3 dimensional flat Euclidean space R_3 or a 3 dimensional hyperboloid H_3. We will, however, only consider the case that the boundary is S_3 since the results for other cases are almost equivalent.

Motivated by (5), (11), (13) and (15), one assumes the metric of 5 dimensional space time as follows:

$$ds^2 = dy^2 + {\rm e}^{2A(y,\sigma)}\tilde g_{\mu\nu}dx^\mu dx^\nu \ , \quad \tilde g_{\mu\nu}dx^\mu dx^\nu \equiv l^2\left(d\sigma^2 + d\Omega_3^2\right) \tag{16}$$

and one identifies A and $\tilde g$ in (16) with those in (7). Then $\tilde F = \tilde G = 0$, $\tilde R = \frac{6}{l^2}$ etc. Due to the assumption (16), the actions in (6) have the following forms:

$$\begin{aligned}
S_{\rm EH} &= \frac{l^4 V_3}{16\pi G}\int dy d\sigma \left\{\left(-8\partial_y^2 A - 20(\partial_y A)^2\right){\rm e}^{4A}\right. \\
&\quad \left. + \left(-6\partial_\sigma^2 A - 6(\partial_\sigma A)^2 + 6\right){\rm e}^{2A} + \frac{12}{l^2}{\rm e}^{4A}\right\} \\
S_{\rm GH} &= \frac{l^4 V_3}{2\pi G}\int d\sigma {\rm e}^{4A}\partial_y A \ , \quad S_1 = -\frac{3l^3 V_3}{8\pi G}\int d\sigma {\rm e}^{4A} \\
\mathcal{W} &= V_3\int d\sigma\left[b' A\left(2\partial_\sigma^4 A - 8\partial_\sigma^2 A\right) - 2(b+b')\left(1 - \partial_\sigma^2 A - (\partial_\sigma A)^2\right)^2\right] \ .
\end{aligned} \tag{17}$$

Here V_3 is the volume or area of the unit 3 sphere.

In the bulk, one obtains the following equation of motion from $S_{\rm EH}$ by the variation over A:

$$0 = \left(-24\partial_y^2 A - 48(\partial_y A)^2 + \frac{48}{l^2}\right){\rm e}^{4A} + \frac{1}{l^2}\left(-12\partial_\sigma^2 A - 12(\partial_\sigma A)^2 + 12\right){\rm e}^{2A} \ , \tag{18}$$

which corresponds to one of the Einstein equations. Then one finds solutions, A_S, which corresponds to the metric of AdS$_5$ in (5) with (11), A_E, which corresponds to (13), and A_H, which corresponds to (15).

$$\begin{aligned}
&A = A_S,\ A_E,\ A_H, \\
&A_S = \ln\sinh\frac{y}{l} - \ln\cosh\sigma \ , \quad A_E = \frac{y}{l} + \sigma \ , \quad A_H = \ln\cosh\frac{y}{l} - \ln\sinh\sigma \ .
\end{aligned} \tag{19}$$

One should note that all the metrics in (19) locally describe the same spacetime, that is the local region of AdS_5, in the bulk. As we assume, however, that there is a brane at $y = y_0$, the shapes of the branes are different from each other due to the choice of the metric.

On the brane at the boundary, one gets the following equation:[14]

$$\begin{aligned} 0 &= \frac{48 l^4}{16\pi G}\left(\partial_y A - \frac{1}{l}\right)\mathrm{e}^{4A} + b'\left(4\partial_\sigma^4 A - 16\partial_\sigma^2 A\right) \\ &\quad -4(b+b')\left(\partial_\sigma^4 A + 2\partial_\sigma^2 A - 6(\partial_\sigma A)^2\partial_\sigma^2 A\right) . \end{aligned} \tag{20}$$

We should note that the contributions from S_{EH} and S_{GH} are twice from the naive values since we have two bulk regions which are connected with each other by the brane. Substituting the bulk solution $A = A_S$ in (19) into (20) and defining the radius R of the brane by $R \equiv l \sinh \frac{y_0}{l}$, one obtains

$$0 = \frac{1}{\pi G}\left(\frac{1}{R}\sqrt{1 + \frac{R^2}{l^2}} - \frac{1}{l}\right) R^4 + 8b' . \tag{21}$$

Note that eq.(21) does not depend on b. This equation generalizes the corresponding result of ref.[1] for the case when the arbitrary amount of quantum conformal matter sits on de Sitter wall. Adopting AdS/CFT correspondence one can argue that in symmetric phase the quantum brane matter comes due to maximally SUSY Yang-Mills theory.

As we have $b' \to -\frac{N^2}{4(4\pi)^2}$ in case of the large N limit of $\mathcal{N} = 4$ $SU(N)$ SYM theory, we find

$$\frac{R^3}{l^3}\sqrt{1 + \frac{R^2}{l^2}} = \frac{R^4}{l^4} + \frac{GN^2}{8\pi l^3} , \tag{22}$$

which exactly coincides with the result in [1]. This equation has the unique solution for positive radius which defines brane-world de Sitter universe (inflation) induced by quantum effects.

On the other hand, if one substitutes the solution $A = A_E$ in (19), corresponding to flat Euclidean brane into (20), we find that (20) is always (independent of y_0) satisfied since $\partial_y A = \frac{1}{l}$ and $\partial_\sigma^2 A = 0$.

If one substitutes $A = A_H$ in (19), which corresponds to the brane with the shape of the hyperboloid, and one defines the radius R_{H} of the brane by $R_{\mathrm{H}} \equiv l \cosh \frac{y_0}{l}$, then

$$0 = \frac{1}{\pi G}\left(\pm\frac{1}{R_{\mathrm{H}}}\sqrt{-1 + \frac{{R_{\mathrm{H}}}^2}{l^2}} - \frac{1}{l}\right) {R_{\mathrm{H}}}^4 + 8b' . \tag{23}$$

We should note that eq.(23) does not depend on b again. In order that Eq.(23) has a solution, b' must be positive, which conflicts with the case of $\mathcal{N} = 4$ $SU(N)$ SYM theory

[14] We should note that the radial (y) component of the geodesic equation for the in the metric (16) is given by $\frac{d^2 x^y}{d\tau^2} + \partial_y A \mathrm{e}^{2A}\left(\frac{dx^t}{d\tau}\right)^2 = 0$. Here τ is the proper time and one can normalize $\mathrm{e}^{2A}\left(\frac{dx^t}{d\tau}\right)^2 = 1$ and obtain $\frac{d^2 x^y}{d\tau^2} + \partial_y A = 0$. Therefore the classical part in (20) expresses the balance of the gravitational force and tension. The mass density of the brane is given by $\frac{3}{8\pi G}$.

or usual conformal matter. In general, however, for some exotic theories, like higher derivative conformal scalar[15], b' can be positive and one can assume for the moment that $b' > 0$ here. Hence, we showed that quantum, conformally invariant matter on the wall, leads to the inducing of inflationary 4d de Sitter-brane universe realized within 5d Anti-de Sitter space (a la Randall-Sundrum[19, 2]). Of course, analytical continuation of our 4d sphere to Lorentzian signature is supposed what leads to ever expanding inflationary brane-world universe. In 4d QFT (no higher dimensions) such idea of anomaly induced inflation has been suggested long ago in refs.[26]. On the same time the inducing of 4d hyperbolic wall in brane universe is highly suppressed and may be realized only for exotic conformal matter. The analysis of the role of domain wall CFT to metric fluctuations may be taken from results of ref.[1].

It is interesting to note that our approach is quite general. In particulary, it is not difficult to take into account the quantum gravity effects (at least, on the domain wall). That can be done by using the corresponding analogs of central charge for various QGs which may be taken from beta-functions listed in book [24]. In other words, there will be only QG contributions to coefficients b, b' but no more changes in subsequent equations.

Generalizations to the cases of the gravity coupled with dilaton are given in [11, 22]. Especially in [22], supersymmetric cases are discussed. This will be considered below.

4 Classical supersymmetric brane-world

The 5d $\mathcal{N} = 8$ gauged supergravity can be obtained from 10d IIB supergravity, where the spacetime is compactified into $\mathrm{S}_5 \times \mathrm{M}_5$. Here S_5 is 5d sphere and M_5 is a 5d manifold, where gauged supergravity lives. The bosonic sector (gravity and scalar part) of the 5d gauged supergravity is given by

$$S_{\rm bulk} = \frac{1}{16\pi G}\int d^5x\sqrt{-g_{(5)}}\left(R_{(5)} - \frac{1}{2}g_{ij}(\phi_k)\nabla_\mu\phi^i\nabla^\mu\phi^j - V(\phi^i)\right) . \tag{24}$$

In the 5-dimensional maximal supergravity, the scalar field parametrizes the coset of $E_6/SL(6,R)$. In (24), $g_{ij}(\phi^k)$ is the induced scalars metric and the potential $V(\phi^i)$ can be rewritten in terms of the superpotential $W(\phi_i)$:[16]

$$V(\phi_i) = \frac{3}{4}\left(\frac{3}{2}g^{ij}(\phi_k)\frac{dW(\phi_k)}{d\phi^i}\frac{dW(\phi_k)}{d\phi^j} - W(\phi_k)^2\right) . \tag{25}$$

We choose the boundary action $S_{\rm bndry}$ in the following form:

$$S_{\rm bndry} = \mp\frac{3}{16\pi G}\int d^4x\sqrt{-g_{(4)}}W(\phi) . \tag{26}$$

This tells that the brane is BPS saturated state, that is, the brane preserves the half of the supersymmetries of the whole system. In order to see it, one considers the simple

[15]Such higher derivative conformal scalar naturally appears in infra-red sector of quantum gravity[25].

[16]Eq.(25) can be regarded as the definition of $W(\phi_i)$ for rather general potential $V(\phi)$ even if there is no supersymmetry. The corresponding potential in case of $D = 5$ $\mathcal{N} = 8$ supergravity, including higher rank tensors, was found in [27]. The potential under discussion may be considered as the one corresponding to some its subsector, for example, as the one discussed in last of refs.[9].

case that only one scalar field ϕ is non-trivial and $g_{\phi\phi} = 1$ and investigate the equations of motion given by the simplified action:

$$S = \frac{1}{16\pi G}\left[\int_M d^5x\sqrt{-g_{(5)}}\left(R_{(5)} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) - \sum_{i=1,2}\int_{B_i} d^4x\sqrt{-g_{(4)}}U_i(\phi)\right] . \tag{27}$$

Here B_i's express the boundaries or branes. This model generalizes that in [19], where two branes are boundaries of the bulk spacetime. The model with only one brane can be obtained, of course by letting one of the branes go to infinity. At first, we do not specify the form of $U_i(\phi)$ but by investigating the equations of motion, we will see the correspondence with (26). Assume the metric in 5d spacetime as

$$ds^2 = dz^2 + \mathrm{e}^{2A(z)}\eta_{ij}dx^i dx^j , \tag{28}$$

and ϕ only depends on z. One also supposes the branes sit on $z = z_1$ and $z = z_2$, respectively. Then the equations of motion are given by

$$\phi'' + 4A'\phi' = \frac{dV}{d\phi} + \sum_{i=1,2}\frac{dU_i(\phi)}{d\phi}\delta(z - z_i) , \tag{29}$$

$$4A'' + 4(A')^2 + \frac{1}{2}(\phi')^2 = -\frac{V}{3} - \frac{2}{3}\sum_{i=1,2}U_i(\phi)\delta(z - z_i) , \tag{30}$$

$$A'' + 4(A')^2 = -\frac{V}{3} - \frac{1}{6}\sum_{i=1,2}U_i(\phi)\delta(z - z_i) . \tag{31}$$

Here $' \equiv \frac{d}{dz}$. For purely bulk sector ($z_1 < z < z_2$, as we assume $z_1 < z_2$), Eqs. (29-31) have the following first integrals:

$$\phi' = \frac{3}{2}\frac{dW}{d\phi} , \quad A' = -\frac{1}{4}W . \tag{32}$$

(Here the ambiguity in the sign when solving Eq.(25) with respect to $W(\phi)$ is absorbed into the definition of $W(\phi)$.) One should note that classical solutions do not always satisfy the above Eqs. in (32). Classical solutions are generally not invariant under the supersymmetry transformations and the supersymmetry in the bulk is broken in the classical background. Eq.(32) is nothing but the condition that the classical solution is invariant under the half of the supersymmetry transformations. When there are branes, any solution of the equations of motion including the equation coming from the branes might not satisfy Eqs.(32). We now investigate the condition for the brane action which allows solution satisfying Eqs.(32). Then some of the supersymmetries are preserved in the whole system.

Near the branes, Eqs.(29-31) have the forms $\phi'' \sim \frac{dU_i(\phi)}{d\phi}\delta(z - z_i)$, $A'' \sim -\frac{U_i(\phi)}{6}\delta(z - z_i)$. Then

$$2\phi' \sim \frac{dU_i(\phi)}{d\phi}\mathrm{sgn}(z - z_i) , \quad 2A' \sim -\frac{U_i(\phi)}{6}\mathrm{sgn}(z - z_i) , \tag{33}$$

at $z = z_i$.[17] If Eqs.(32) are satisfied, one finds

$$U_1(\phi) = 3W(\phi) \ , \quad U_2(\phi) = -3W(\phi) \ . \tag{34}$$

Eq.(34) reproduces Eq.(26). Note that Eq.(32) is nothing but the BPS condition, where the half of the supersymmetries in the whole system are preserved. As we are considering the solution where fermionic fields vanish, the variations of the fermionic fields under the supersymmetry transformation should vanish if the solution preserves the supersymmetry. If Eq.(32) is satisfied, the variations of gravitino and dilatino vanish under the half of the supersymmetry transformation.

As an extension, one can consider the case that the brane is curved. Instead of (28), we take the following metric:

$$ds^2 = dz^2 + \mathrm{e}^{2A(z)} \tilde{g}_{ij} dx^i dx^j \ , \tag{35}$$

Here $\tilde{g}_{ij}$ is the metric of the Einstein manifold, which is defined by

$$\tilde{R}_{ij} = k\tilde{g}_{ij} \ , \tag{36}$$

where $\tilde{R}_{ij}$ is the Ricci tensor given by $\tilde{g}_{ij}$ and k is a constant. Then Eqs.(29) and (30) do no change but one obtains the following equation instead of (31):

$$A'' + 4(A')^2 = k\mathrm{e}^{2A} - \frac{V}{3} - \frac{1}{6} \sum_{i=1,2} U_i(\phi)\delta(z - z_i) \ . \tag{37}$$

Especialy when $k = 0$, one gets the previous solution for ϕ, A and U_i. Even for $k = 0$, the brane is not always flat, for example, if as $\tilde{g}_{ij}$ in (35), we choose the metric of the Schwarzschild black hole or Kerr black hole spacetime, then Eq.(36) is satisfied since the Ricci tensor vanishes.

Therefore the brane solutions with these black holes of $k = 0$ would preserve the supersymmetry of the whole system. When $k \neq 0$, however, one finds that Eq.(37) has no solution which satisfies the BPS condition (32). This might tell that classical curved brane breaks the supersymmetry in such formalism. When $k > 0$, the brane is 4d de Sitter space or 4d sphere when Wick-rotated to the Euclidean signature. On the other hand, when $k < 0$, the brane is 4d anti-de Sitter space or 4d hyperboloid in the Euclidean signature.

5 Supersymmetric brane new world

If 10d spacetime, where IIB supergraviry lives, is compactified into $\mathrm{S}_5 \times \mathrm{M}_5$, we effectively obtain 5d $\mathcal{N} = 8$ gauged supergravity in the bulk and 4d $\mathcal{N} = 4$ $SU(N)$ or $U(N)$

[17] Here the function $\mathrm{sgn}(x)$ is defined by

$$\mathrm{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

and z is the coordinate perpendicular to the boundary or brane.

super-Yang-Mills theory coupled with (super)gravity on the brane. On the other hand, if 10d spacetime is compactified into $X_5 \times M_5$, where X_5 is S_5/Z_2, $\mathcal{N}=2$ $Sp(N)$ super-Yang-Mills theory coupled with (super)gravity would be realized on the brane. Since the matter multiplets of the super-Yang-Mills are coupled with (super)gravity, they generate a conformal anomaly on quantum level.

In this section, we include the trace anomaly induced action on the brane to the analysis of supersymmetric brane-world. One chooses the brane action to preserve the supersymmetry as in the previous section and consider the solution where the scalar field is non-trivial. Here mainly Euclidean signature is used.

As curved brane is considered, we assume that the metric of (Euclidean) AdS has the following form:

$$ds^2 = dz^2 + \sum_{i,j=1}^{4} g_{(4)ij} dx^i dx^j \ , \quad g_{(4)ij} = \mathrm{e}^{2\tilde{A}(z)} \hat{g}_{ij} \ . \tag{38}$$

Here $\hat{g}_{ij}$ is the metric of the Einstein manifold as in (36). One can consider two copies of the regions given by $z < z_0$ and glue two regions putting a brane at $z = z_0$.

Let us start with Euclidean signature action S which is sum of the Einstein-Hilbert action $S_{\rm EH}$ with kinetic term and potential $V(\phi)$ for dilaton ϕ, the Gibbons-Hawking surface term $S_{\rm GH}$, the surface counterterm S_1 and the trace anomaly induced action $\mathcal{W}$ [11]:

$$\begin{aligned}
&S = S_{\rm EH}^\phi + S_{\rm GH} + 2S_1^\phi + \tilde{\mathcal{W}}, \\
&S_{\rm EH}^\phi = \frac{1}{16\pi G}\int d^5x \sqrt{g_{(5)}} \left(R_{(5)} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right), \\
&S_{\rm GH} = \frac{1}{8\pi G}\int d^4x \sqrt{g_{(4)}} \nabla_\mu n^\mu \ , \quad S_1^\phi = -\frac{3}{16\pi G l}\int d^4x \sqrt{g_{(4)}} W(\phi), \\
&\mathcal{W}^\phi = \mathcal{W} + C\int d^4x \sqrt{\tilde{g}} A\phi \left[\widetilde{\Box}^2 + 2\widetilde{R}_{\mu\nu}\widetilde{\nabla}_\mu\widetilde{\nabla}_\nu - \frac{2}{3}\widetilde{R}\,\widetilde{\Box}^2 + \frac{1}{3}(\widetilde{\nabla}^\mu \widetilde{R})\widetilde{\nabla}_\mu\right]\phi \ .
\end{aligned} \tag{39}$$

Here the quantities in the 5 dimensional bulk spacetime are specified by the suffices $_{(5)}$ and those in the boundary 4 dimensional spacetime are specified by $_{(4)}$. $\mathcal{W}$ in (39) is defined in (17).

In [11], as an action on the brane, corresponding to S_1^ϕ in (39), the action motivated by the counterterm method in AdS/CFT correspondence was used:

$$S_1^{\rm NOO} = -\frac{1}{16\pi G l}\int d^4x \sqrt{g_{(4)}} \left(\frac{6}{l} + \frac{l}{4}\Phi(\phi)\right) \ . \tag{40}$$

In the AdS/CFT correspondence, the divergence coming from the infinite volume of AdS corresponds to the UV divergence in the CFT side. The counterterm which cancells the leading divergence in the AdS side corresponds to the above action $S_1^{\rm NOO}$.

In the present framework, the spacetime inside the brane has finite volume and there might be ambiguities when choosing the counterterm. Here we give S_1 in (39) in terms of the superpotential $W(\phi)$ corresponding to (25), which is given by

$$V(\phi) = \frac{3}{4}\left(\frac{3}{2}\left(\frac{dW(\phi)}{d\phi}\right)^2 - W(\phi)^2\right) \ . \tag{41}$$

This is natural in terms of supersymmetric extension [9] of the Randall-Sundrum model [19, 2]. This action tells that the brane is BPS saturated state and the half of the supersymmetries could be conserved [8]. The factor 2 in front of S_1 in (39) is coming from that we have two bulk regions which are connected with each other by the brane.

In (39), one chooses the 4 dimensional boundary metric as in (7). We should distinguish A and $\tilde{g}_{\mu\nu}$ with $\tilde{A}(z)$ and $\hat{g}_{ij}$ in (38). We also specify the quantities given by $\tilde{g}_{\mu\nu}$ by using $\tilde{}$.

Let us start the consideration of field equations. It is often convienient that one assumes the metric of 5 dimensional spacetime as follows:

$$ds^2 = g_{(5)\mu\nu}dx^\mu dx^\nu = f(y)dy^2 + y\sum_{i,j=1}^{4}\hat{g}_{ij}(x^k)dx^i dx^j. \tag{42}$$

Here $\hat{g}_{ij}$ is the metric of 4 dimensional Einstein manifold as in (36). A coordinate corresponding to z in (38) can be obtained by $z = \int dy\sqrt{f(y)}$, and solves y with respect to z. Then the warp factor is $\mathrm{e}^{2\hat{A}(z,k)} = y(z)$.

On the brane, we obtain the following equations:

$$\begin{aligned}
0 &= \frac{48l^4}{16\pi G}\left(\partial_z A - \frac{1}{2}W(\phi)\right)\mathrm{e}^{4A} + b'\left(4\partial_\sigma^4 A - 16\partial_\sigma^2 A\right) \\
&\quad -4(b+b')\left(\partial_\sigma^4 A + 2\partial_\sigma^2 A - 6(\partial_\sigma A)^2\partial_\sigma^2 A\right) + 2C\left(\partial_\sigma^4\phi - 4\partial_\sigma^2\phi\right), \qquad (43) \\
0 &= -\frac{l^4}{8\pi G}\mathrm{e}^{4A}\partial_z\phi - \frac{3l^3\mathrm{e}^{4A}}{8\pi G}\frac{dW(\phi)}{d\phi} + C\left\{A\left(\partial_\sigma^4\phi - 4\partial_\sigma^2\phi\right) + \partial_\sigma^4(A\phi) - 4\partial_\sigma^2(A\phi)\right\} \qquad (44)
\end{aligned}$$

In (43) and (44), using the coordinate z and choosing $l^2\mathrm{e}^{2\hat{A}(z,k)} = y(z)$ one uses the form of the metric as in (16). Then

$$\begin{aligned}
&A(z,\sigma) = \hat{A}(z,k=3) - \ln\cosh\sigma\ , \quad A(z,\sigma) = \hat{A}(z,k=0) + \sigma\ , \\
&A(z,\sigma) = \hat{A}(z,k=-3) - \ln\sinh\sigma\ ,
\end{aligned} \tag{45}$$

for the unit sphere ($k=3$), for the flat Euclidean space ($k=0$), and for the unit hyperboloid ($k=-3$), respectively. We now identify A and $\tilde{g}$ in (16) with those in (7). Then one finds $\tilde{F} = \tilde{G} = 0$, $\tilde{R} = \frac{6}{l^2}$ etc. Note that the sphere corresponds to de Sitter space and the hyperboloid to anti-de Sitter space when we Wick-rotate the Euclidean signature to the Lorentzian one.

By using the equations of motions in the bulk given by the Einstein action (39), one can obtain an equation that contains only the dilaton field ϕ (and, of course, bulk potential):

$$\begin{aligned}
0 &= \left\{\frac{5k}{2} - \frac{k}{4}y^2\left(\frac{d\phi}{dy}\right)^2 + \left(\frac{3}{2}y + \frac{y^3}{6}\left(\frac{d\phi}{dy}\right)^2\right)\frac{V(\phi)}{2}\right\}\frac{d\phi}{dy} \\
&\quad + \frac{y^2}{2}\left(\frac{2k}{y} - \frac{1}{2}V(\phi)\right)\frac{d^2\phi}{dy^2} - \left(\frac{3}{4} - \frac{y^2}{8}\left(\frac{d\phi}{dy}\right)^2\right)\frac{dV(\phi)}{d\phi}\ .
\end{aligned} \tag{46}$$

Several solutions have been found in third ref.[11] by assuming the dilaton and bulk potentials as:

$$\phi(y) = p_1\ln(p_2 y)\ , \quad -V(\phi) = c_1\exp(a\phi) + c_2\exp(2a\phi)\ , \tag{47}$$

where a, p_1, p_2, c_1, c_2 are some constants.

When $k \neq 0$, a special solution is given by

$$c_1 = \frac{6kp_2p_1^2}{3-2p_1^2} \ , \quad c_2 = 0 \ , \quad a = -\frac{1}{p_1} \ , \quad p_1 \neq \pm\sqrt{6} \ , \quad f(y) = \frac{3-2p_1^2}{4ky} \ . \tag{48}$$

Here the superpotential $W(\phi)$ is given by

$$W(\phi) = 8p_1^2\sqrt{\frac{p_2 k}{(3-2p_1^2)(8p_1^2-3)}}\mathrm{e}^{-\frac{\phi}{2p_1}} \ . \tag{49}$$

The potential (47) with the coefficients c_1 and c_2 in (48) corresponds to special RG flow in 5d $\mathcal{N}=8$ gauged supergravity where only one scalar from 42 scalars is considered. If we define q^2 by

$$q^2 \equiv \frac{4k}{3-2p_1^2} > 0 \ , \tag{50}$$

the solution when $k=0$ can be obtained by taking $k \to 0$ limit and keeping q^2 finite. In the limit, Eqs.(48) and (49) have the following forms:

$$c_1 = \frac{9}{4}q^2 p_2 \ , \quad c_2 = 0 \ , \quad a = -\frac{1}{p_1} \ , \quad p_1 \neq \pm\sqrt{6} \ , \quad f(y) = \frac{1}{q^2 y} \tag{51}$$

$$W(\phi) = 2\sqrt{q^2 p_2}\mathrm{e}^{-\phi\sqrt{\frac{3}{2}}} \ . \tag{52}$$

This solution satisfies Eq.(32), which is the BPS condition, i.e., the solution preserves the half of the supersymmetries in the bulk space.

The solutions in (48) and (51) have a singularity at $y=0$. In fact the scalar curvature $R_{(5)}$ is given $R_{(5)} = -\frac{3}{2}\frac{p_1^2 q^2}{y}$. Here we assume q^2 is defined by (50) even if $k \neq 0$. When $k=3$, the brane becomes de Sitter space after the Wick-rotation. Then $y=0$ corresponds to the horizon in the bulk 5d space. Therefore in $k=3$ case, the singularity is not exactly naked.

In the coordinate system (42), brane Eq.(44) has the following form:

$$0 = -\frac{y_0^2}{8\pi G\sqrt{f(y_0)}}\partial_y\phi - \frac{3y_0^2}{8\pi G}\frac{dW(\phi_0)}{d\phi} + 6C\phi_0 \ . \tag{53}$$

Here ϕ_0 ($\tilde{\phi}_0$) is the value of the dilaton ϕ on the brane. Eq.(43) has the following form:

$$0 = \tfrac{3y_0^2}{16\pi G}\left(\tfrac{1}{2y_0\sqrt{f(y_0)}} - \tfrac{l}{2}W(\phi_0)\right) + 8b' \qquad \text{for } k \neq 0 \text{ case} \tag{54}$$

$$0 = \tfrac{3y_0^2}{16\pi G}\left(\tfrac{1}{2y_0\sqrt{f(y_0)}} - \tfrac{l}{2}W(\phi_0)\right) \qquad \text{for } k = 0 \text{ case.} \tag{55}$$

When $k=0$, where $p_1^2 = \frac{3}{2}$, the second equation in (54) is satisfied trivially but the first equation has the following form:

$$-\frac{q y_0^{\frac{3}{2}}}{8\pi G}\sqrt{\frac{3}{2}} = 6C\phi_0 \ . \tag{56}$$

Then the value ϕ_0 of the dilaton on the brane depends on y_0. We should note that the obtained solution for $k=0$ is really supersymmetric in the whole system since the corresponding bulk solution (51) satisfies the BPS condition Eq.(32) which tells the solution preserves the half of the supersymmetries in the bulk space and the brane action has been chosen not to break the supersymmetry on the brane. It is interesting that even in case of $k=0$, the quantum effect is included in (56) through the parameter C (coefficient of dilatonic term in conformal anomaly). In the classical case, where $C=0$, the value of the scalar field ϕ_0 is a free parameter. Quantum effects suggest the way for dynamical determination of brane dilaton.

When $k \neq 0$, by substituting the solution in (48) into (53) and (54), one finds

$$\frac{p_1 y_0^{\frac{3}{2}}}{4\pi G}\sqrt{\frac{k}{3-2p_1^2}}\left(1-\frac{6}{\sqrt{8p_1^2-3}}\right) = 6C\phi_0 \tag{57}$$

$$\frac{3y_0^{\frac{3}{2}}}{16\pi G}\sqrt{\frac{k}{3-2p_1^2}}\left(1-\frac{2p_1^2}{\sqrt{8p_1^2-3}}\right) = -8b' \ . \tag{58}$$

Since $b'<0$, Eqs.(57) and (58) have non-trivial solutions for ϕ_0 and y_0 if

$$\frac{k}{3-2p_1^2} > 0 \quad \text{and} \quad \frac{1}{2} < p_1^2 < \frac{3}{2} \ . \tag{59}$$

When $k=3$, where the brane is 4d sphere (de Sitter space when we Wick-rotate the brane metric to Lorentzian signature), we have

$$\frac{3}{2} > p_1^2 > \frac{7}{8} \ . \tag{60}$$

On the other hand, if $k=-3$, where the brane is 4d hyperboloid (anti-de Sitter after the Wick-rotation), there is no solution since the conditions in (59) conflict with each other. Note that de Sitter brane ($k>0$) solution does not exist on the classical level but the solution appeared after inclusion of the quantum effects of brane matter in accordance with AdS/CFT.

If Eq.(60) is satisfied, Eqs.(57) and (58) can be explicitly solved with respect to y_0 and ϕ_0. This situation is very different from non-supersymmetric case in [11], where S_1 was chosen as in (40). In third ref. from [11], it was very difficult to solve the equations corresponding to (57) and (58), explicitly. This indicates that supersymmetry simplifies the situation and the approach we adopt is right way to construct supersymmetric new brane world. Moreover, quantum effects may give a natural mechanism for SUSY breaking.

If one writes $y_0 = R_b^2$, R_b corresponds to the radius of the sphere ($k=3$). Since $b' \propto N^2$ for large N, from Eqs.(4) and (58), one gets

$$R_b \propto \left(GN^2\right)^{\frac{1}{3}} \sim N^{\frac{3}{4}} \ . \tag{61}$$

Despite some modern progress, it is clear that much work is necessary in order to construct consistent supersymmetric New Brane World.

6 Gravity perturbations

It is known that brane gravity trapping occurs on curved brane in a different way than on flat brane. For example, in refs.[28, 29] the AdS_4 branes in AdS_5 were discussed and the existence of the massive normalizable mode of graviton was found. In these papers, the tensions of the branes are free parameters but in the case treated in the present section the tension is dynamically determined. As brane solutions are found in the previous section when the brane is flat or de Sitter space, it is reasonable to consider perturbation around the solution.

Let us regard the brane as an object with a tension $\tilde{U}(\phi)$ and assume the brane can be effectively described by the following action, as in (27) (for simplicity, we only consider the brane corresponding to $i = 2$, or the limit $z_1 \to -\infty$):

$$S_{\rm brane} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g_{(4)}}\tilde{U}(\phi) \ . \tag{62}$$

Then using the Einstein equation as in (31), one finds

$$A'' + 4(A')^2 = -\frac{V}{3} - \frac{\tilde{U}(\phi)}{6}\delta(z - z_0) \ . \tag{63}$$

Here we assume that there is a brane at $z = z_0 (= z_2)$. Then at $z = z_0$

$$A'|_{z=z_0} = -\frac{\tilde{U}(\phi)}{6} \ . \tag{64}$$

Comparing (64) with (43) etc. one gets, when $k \neq 0$

$$\tilde{U}(\phi) = -\frac{3}{l}W(\phi_0) + \frac{48\pi G b'}{R_b^4} \ . \tag{65}$$

and when $k = 0$

$$\tilde{U}(\phi) = -\frac{3}{l}W(\phi_0) \ . \tag{66}$$

Note that tension becomes R_b dependent due to the quantum correction when $k \neq 0$, as $b' \sim \mathcal{O}\left(N^2\right)$ and $R_b^4 \sim \mathcal{O}\left(N^3\right)$ from (61), the tension depends on N as $\tilde{U}(\phi) + \frac{3}{l}W(\phi_0) \sim \mathcal{O}\left(N^{-\frac{9}{4}}\right)$. One can understand that the r.h.s. in (66) and the first term in the r.h.s. in (65) are determined from the supersymmetry.

Consider the perturbation by assuming the metric in the following form:

$$ds^2 = \mathrm{e}^{2\hat{A}(\zeta)}\left(d\zeta^2 + \left(\hat{g}_{\mu\nu} + \mathrm{e}^{-\frac{3}{2}\hat{A}(\zeta)}h_{\mu\nu}\right)dx^\mu dx^\nu\right) \ . \tag{67}$$

Here the gauge conditions $h^\mu_{\ \mu} = 0$ and $\nabla^\mu h_{\mu\nu} = 0$ are chosen. Then one obtains the following equation

$$\left(-\partial_\zeta^2 + \frac{9}{4}\left(\partial_\zeta\hat{A}\right)^2 + \frac{3}{2}\partial_\zeta^2\hat{A}\right)h_{\mu\nu} = m^2 h_{\mu\nu} \tag{68}$$

Here m^2 corresponds to the mass of the graviton on the brane

$$\left(\hat{\Box} + \frac{1}{R_b^2}\right)h_{\mu\nu} = m^2 h_{\mu\nu} \ . \tag{69}$$

for $k > 0$ and

$$\hat{\Box} h_{\mu\nu} = m^2 h_{\mu\nu} \tag{70}$$

for $k = 0$. Here $\hat{\Box}$ is 4 dimensional d'Alembertian constructed on $\hat{g}_{\mu\nu}$. Since

$$\pm \mathrm{e}^{A} d\zeta = dz = \sqrt{f} dy \ , \quad \mathrm{e}^{A} = \frac{\sqrt{y}}{l} \ , \tag{71}$$

one finds

$$\pm\zeta = \int dy \sqrt{\frac{f(y)}{y}} \ . \tag{72}$$

If we choose $\zeta = 0$ when $y = y_0$, Eq.(72) for the solution in (48) or (51) gives

$$|\zeta| = -\frac{1}{q} \ln y + \frac{1}{q} \ln y_0 \ . \tag{73}$$

Here we assume q is defined by (50) even if $k > 0$. Since only the square of q is defined in (50), we can choose q to be positive.

Note that brane separates two bulk regions corresponding to $\zeta < 0$ and $\zeta > 0$, respectively. Since y takes the value in $[0, y_0]$, ζ takes the value in $[-\infty, \infty]$. Since $A = \frac{1}{2}\ln y$, from (68) one gets

$$\left(-\partial_\zeta^2 + \frac{9q^2}{4} - 3q\delta(\zeta) \right) h_{\mu\nu} = m^2 h_{\mu\nu} \tag{74}$$

Zero mode solution with m^2 of (74) is given by

$$h_{\mu\nu} = h^{(0)}_{\mu\nu} \mathrm{e}^{-\frac{3q}{2}|\zeta|} \ . \tag{75}$$

Here $h^{(0)}_{\mu\nu}$ is a constant. Any other normalizable solution does not exist. When

$$m^2 > \frac{9}{4} q^2 \ , \tag{76}$$

there are non-normalizable solutions given by

$$h_{\mu\nu} = a_{\mu\nu} \cos\left(|\zeta| \sqrt{m^2 - \frac{9}{4}q^2} \right) + b_{\mu\nu} \sin\left(|\zeta| \sqrt{m^2 - \frac{9}{4}q^2} \right) \ . \tag{77}$$

The coefficients $a_{\mu\nu}$ and $b_{\mu\nu}$ are constants of the integration and they are determined to satisfy the boundary condition, which comes from the δ-function potential in (74),

$$\left. \frac{\partial_\zeta h_{\mu\nu}}{h_{\mu\nu}} \right|_{\zeta \to 0+} = -\frac{3}{2} q \ . \tag{78}$$

Note that zero mode solution (75) satisfies this boundary condition (78). By using (78), we can determine the coefficients $a_{\mu\nu}$ and $b_{\mu\nu}$ for non-normalizable solutions as follows:

$$a_{\mu\nu} = -\frac{2b_{\mu\nu}}{3q} \sqrt{m^2 - \frac{9}{4}q^2} \ . \tag{79}$$

It might be interesting that there is the minimum (76) in the mass of non-normalizable mode. This situation is different from the original Randall-Sundrum model [2]. Although de Sitter brane appears when we include the quantum correction, the minimum itself does not depend on the parameter of the quantum correction b' or N.

Since there is only one normalizable solution corresponding to zero mode (75) and other solutions (77) are non-normalizable, gravity should be localized on the brane and the leading long range potential between two massive sources on the brane should obey the Coulomb law, i.e., $\mathcal{O}\left(r^{-1}\right)$. Here r is the distance between the above two massive sources. Furthermore the existence of the the minimum (76) in the mass of non-normalizable mode indicates that the correction to the Coulomb law should be small.

7 AdS/CFT and quantum-corrected brane entropy

In the previous sections, we have only considered the case that the radius of the brane is constant. In this section, the situation that radius depends on the "time" coordinate is discussed. If we consider the AdS-Schwarzschild background, the obtained equation, which describes the dynamics of the brane, can be regarded as the Friedmann-Robertson-Walker (FRW) equation. ¿From the equation, one obtains the quantum-corrected brane entropy as well as Hubble constant and Hawking temperature.

Let us start with the Minkowski signature action. Then being in the Brane New World, we have the following equation which generalizes the classical brane equation of the motion:

$$\begin{aligned} 0 \ = \ & \frac{48l^4}{16\pi G}\left(A_{,z}-\frac{1}{l}\right)\mathrm{e}^{4A}+b'\left(4\partial_\tau^4 A+16\partial_\tau^2 A\right) \\ & -4(b+b')\left(\partial_\tau^4 A-2\partial_\tau^2 A-6(\partial_\tau A)^2\partial_\tau^2 A\right) \ . \end{aligned} \qquad (80)$$

In (80), one uses the form of the metric as

$$ds^2=dz^2+\mathrm{e}^{2A(z,\tau)}\tilde g_{\mu\nu}dx^\mu dx^\nu \ , \quad \tilde g_{\mu\nu}dx^\mu dx^\nu \equiv l^2\left(-d\tau^2+d\Omega_3^2\right) \ . \qquad (81)$$

Here $d\Omega_3^2$ corresponds to the metric of 3 dimensional unit sphere.

As a bulk space, one takes 5d AdS-Schwarzschild space-time, whose metric is given by

$$ds^2_{\mathrm{AdS-S}}=\frac{1}{h(a)}da^2-h(a)dt^2+a^2d\Omega_3^2 \ , \quad h(a)=\frac{a^2}{l^2}+1-\frac{16\pi GM}{3V_3a^2} \ . \qquad (82)$$

Here V_3 is the volume of the unit 3 sphere. If one chooses new coordinates (z,τ) by

$$\frac{\mathrm{e}^{2A}}{h(a)}A_{,z}^2-h(a)t_{,z}^2=1 \ , \quad \frac{\mathrm{e}^{2A}}{h(a)}A_{,z}A_{,\tau}-h(a)t_{,z}t_{,\tau}=0 \ , \quad \frac{\mathrm{e}^{2A}}{h(a)}A_{,\tau}^2-h(a)t_{,\tau}^2=-\mathrm{e}^{2A} \ . \quad (83)$$

the metric takes the form (81). Here $a=l\mathrm{e}^A$. Furthermore choosing a coordinate $\tilde t$ by $d\tilde t=l\mathrm{e}^A d\tau$, the metric on the brane takes FRW form:

$$\mathrm{e}^{2A}\tilde g_{\mu\nu}dx^\mu dx^\nu=-d\tilde t^2+l^2\mathrm{e}^{2A}d\Omega_3^2 \ . \qquad (84)$$

By solving Eqs.(83), we have

$$H^2 = A_{,z}^2 - h e^{-2A} = A_{,z}^2 - \frac{1}{l^2} - \frac{1}{a^2} + \frac{16\pi G M}{3V_3 a^4} \ . \tag{85}$$

Here the Hubble constant H is defined by $H = \frac{dA}{d\tilde{t}}$. Finally, after some algebra one arrives to

$$\begin{aligned}
H^2 &= -\frac{1}{a^2} + \frac{8\pi G_4 \rho}{3} && (86) \\
\rho &= \frac{l}{a}\left[\frac{M}{V_3 a^3} + \frac{3a}{16\pi G}\left[\left[\frac{1}{l} + \frac{\pi G}{3}\left\{-4b'\left(\left(H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2 + 7HH_{,\tilde{t}\tilde{t}}\right.\right.\right.\right.\right.\right. \\
&\quad \left. +18H^2 H_{,\tilde{t}} + 6H^4\right) + \frac{4}{a^2}\left(H_{,\tilde{t}} + H^2\right)\Big) + 4(b+b')\left(\left(H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2\right.\right. \\
&\quad \left.\left.\left.\left.\left.\left. +7HH_{,\tilde{t}\tilde{t}} + 12H^2 H_{,\tilde{t}}\right) - \frac{2}{a^2}\left(H_{,\tilde{t}} + H^2\right)\right)\right\}\right]^2 - \frac{1}{l^2}\right]\right] \ . && (87)
\end{aligned}$$

This can be regarded as the quantum FRW equation of the brane universe. Here 4d Newton constant G_4 is given by

$$G_4 = \frac{2G}{l} \ . \tag{88}$$

Eq.(86) expresses the quantum correction to the corresponding equation in [18]. In fact, if we put $b = b' = 0$, Eq.(86) reduces to the classical one. Further by differentiating Eq.(86) with respect to $\tilde{t}$, we obtain the second FRW equation

$$\begin{aligned}
H_{,\tilde{t}} &= \frac{1}{a^2} - 4\pi G_4(\rho + p) && (89) \\
\rho + p &= \frac{l}{a}\left[\frac{4M}{3V_3 a^3} - \frac{1}{24 l^3 H}\left[\frac{1}{l} + \frac{\pi G}{3}\left\{-4b'\left(\left(H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2 + 7HH_{,\tilde{t}\tilde{t}}\right.\right.\right.\right.\right. \\
&\quad \left. +18H^2 H_{,\tilde{t}} + 6H^4\right) + \frac{4}{a^2}\left(H_{,\tilde{t}} + H^2\right)\Big) \\
&\quad \left.\left.\left. +4(b+b')\left(\left(H_{,\tilde{t}\tilde{t}\tilde{t}} + 4H_{,\tilde{t}}^2 + 7HH_{,\tilde{t}\tilde{t}} + 12H^2 H_{,\tilde{t}}\right) - \frac{2}{a^2}\left(H_{,\tilde{t}} + H^2\right)\right)\right\}\right]\right. \\
&\quad \times\left\{-4b'\left(\left(H_{,\tilde{t}\tilde{t}\tilde{t}\tilde{t}} + 15H_{,\tilde{t}}H_{\tilde{t}\tilde{t}} + 7HH_{,\tilde{t}\tilde{t}\tilde{t}} + 18H^2 H_{,\tilde{t}\tilde{t}} + 36HH_{,\tilde{t}}^2\right.\right.\right. \\
&\quad \left. +24H^3 H_{,\tilde{t}}\right) + \frac{4}{a^2}\left(H_{,\tilde{t}\tilde{t}} - 2H^3\right)\Big) + 4(b+b')\left(\left(H_{,\tilde{t}\tilde{t}\tilde{t}\tilde{t}} + 15H_{,\tilde{t}}H_{,\tilde{t}\tilde{t}}\right.\right. \\
&\quad \left.\left.\left. +7HH_{,\tilde{t}\tilde{t}\tilde{t}} + 12H^2 H_{,\tilde{t}\tilde{t}} + 24HH_{,\tilde{t}}^2\right) - \frac{2}{a^2}\left(H_{,\tilde{t}\tilde{t}} - 2H^2\right)\right)\right\}\Big] \ . && (90)
\end{aligned}$$

The quantum corrections from CFT are included into the definition of energy (pressure). These quantum corrected FRW equations are written from quantum-induced brane-world perspective. Similar equations from the point of view of 4d brane observer (who does not know about 5d AdS bulk) have been presented in ref.[17]. Clearly, brane-world approach gives more information.

Note that when a is large, the metric (82) has the following form:

$$ds_{\mathrm{AdS-S}}^2 \to \frac{a^2}{l^2}\left(dt^2 + l^2 d\Omega_3^2\right) \ , \tag{91}$$

which tells that the CFT time $t_{\rm CFT}$ is equal to the AdS time t times the factor $\frac{a}{l}$: $t_{\rm CFT}=\frac{a}{l}t$. Therefore the energy $E_{\rm CFT}$ in CFT is related with the energy $E_{\rm AdS}$ in AdS by [18]

$$E_{\rm CFT}=\frac{l}{a}E_{\rm AdS}\ . \tag{92}$$

The factor $\frac{l}{a}$ in front of Eqs.(87) and (90) appears due to the above scaling of the energy in (92) or time.

The AdS$_5$-Schwarzschild black hole solution in (82) has a horizon at $a=a_H$, where $h(a)$ vanishes [30]:

$$h(a_H)=\frac{a_H^2}{l^2}+1-\frac{16\pi GM}{3V_3 a_H^2}=0\ . \tag{93}$$

Then considering the moment the brane crosses these points and using (86), one gets

$$\begin{aligned} H &= \pm\left[\frac{1}{l}+\frac{\pi G}{3}\left\{-4b'\left(\left(H_{,\tilde t\tilde t\tilde t}+4H_{,\tilde t}^2+7HH_{,\tilde t\tilde t}+18H^2H_{,\tilde t}+6H^4\right)+\frac{4}{a_H^2}\left(H_{,\tilde t}+H^2\right)\right)\right.\right. \\ &\quad \left.\left. +4(b+b')\left(\left(H_{,\tilde t\tilde t\tilde t}+4H_{,\tilde t}^2+7HH_{,\tilde t\tilde t}+12H^2H_{,\tilde t}\right)-\frac{2}{a_H^2}\left(H_{,\tilde t}+H^2\right)\right)\right\}\right]\ . \end{aligned} \tag{94}$$

The sign $\pm$ depends on whether the brane is expanding or contracting. Obviously, if the higher derivative of the Hubble constant H is large, the quantum correction becomes essential.

We now assume that the brane behaves as de Sitter (inflationary) space $a=A\cosh B\tilde t$ near the horizon. This is quantum-induced brane universe. Note that this is not the solution for positive (non-vanishing) black hole mass $M>0$ but the above assumption is very natural. Then Eqs.(93) and (94) have the following forms:

$$h(a_H) = \frac{A^2\cosh^2\tilde t_H}{l^2}+1-\frac{16\pi GM}{3V_3A^2\cosh^2\tilde t_H}=0 \tag{95}$$

$$\begin{aligned} H &= \pm\left[\frac{1}{l}+\frac{\pi G}{3}\left\{-4b'\left(-4\left(B^4-\frac{B^2}{A^2}\right)\frac{1}{\cosh^2\tilde t_H}+6B^4\right)\right.\right. \\ &\quad \left.\left. +8(b+b')\left(B^4-\frac{B^2}{A^2}\right)\frac{1}{\cosh^2\tilde t_H}\right.\right]\ . \end{aligned} \tag{96}$$

Here the brane crosses the horizon when $\tilde t=\tilde t_H$. Thus, quantum-corrected Hubble parameter at the horizon is defined. The quantum correction becomes large when the rate B of expansion of the universe is large.

Let the entropy $\mathcal{S}$ of CFT on the brane is given by the Bekenstein-Hawking entropy of the AdS$_5$ black hole $\mathcal{S}=\frac{V_H}{4G}$. Here V_H is the area of the horizon, which is equal to the spatial brane when the brane crosses the horizon: $V_H=a_H^3V_3$. If the total entropy $\mathcal{S}$ is constant during the cosmological evolution, the entropy density s is given by (see [18])

$$s=\frac{\mathcal{S}}{a^3V_3}=\frac{la_H^3}{2G_4a^3}\ . \tag{97}$$

Here Eq.(88) is used. The expression in (97) is identical with the classical one. The quantum correction appear when we express s in terms of the quantities in brane universe, say H, $H_{,\tilde t}$ etc., by using (94).

The Hawking temperature of the black hole is given by (see [18])

$$T_H = \frac{h'(a_H)}{4\pi} = \frac{a_H}{2\pi l^2} + \frac{8GM}{3V_3 a_H^3} = \frac{a_H}{\pi l^2} + \frac{1}{2\pi a_H} \ . \tag{98}$$

Here (93) is used. As in (92), the temperature T on the brane is different from that of AdS$_5$ by the factor $\frac{l}{a}$:

$$T = \frac{l}{a} T_H = \frac{a_H}{\pi a l} + \frac{l}{2\pi a a_H} \ . \tag{99}$$

Especially when $a = a_H$

$$T = \frac{1}{\pi l} + \frac{l}{2\pi a_H^2} \ . \tag{100}$$

For the solution in the form of $a = A\cosh B\tilde{t}$, one gets

$$T = \frac{l}{\pi}\left[-H_{,\tilde{t}} \pm \frac{\pi G}{3}(48b' + 16b)\left(B^4 - \frac{B^2}{A^2}\right) \frac{\sinh B\tilde{t}_H}{\cosh^3 B\tilde{t}_H}\right] \ . \tag{101}$$

The quantum correction becomes dominant when $B\tilde{t}_H$ is of order unity but B ($\neq \frac{1}{A}$) is large or A is small. Since the radius of the horizon is given by $a_H = A\cosh B\tilde{t}_H$, this might mean that if quantum correction is large then the radius of the black hole is small.

In [15] it has been pointed out that the first FRW equation (86) can be regarded as the 4-dimensional analogue of the Cardy formula [33] for the entropy $\mathcal{S}$ of the CFT on the brane. In fact identifying

$$\frac{2\pi\rho V a}{3} \Rightarrow 2\pi L_0 \ , \quad \frac{V}{8\pi G_4 a} \Rightarrow \frac{c}{24} \ , \quad \frac{4\pi H V}{8\pi G_4} \Rightarrow \mathcal{S}_H \ , \tag{102}$$

one finds

$$\mathcal{S} = 2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} \ . \tag{103}$$

Furhtermore if we define the Bekenstein entropy S_B and the Bekenstein-Hawking entropy S_{BH} by

$$S_B = \frac{2\pi}{3} E a = 2\pi L_0 \ , \quad S_{BH} = \frac{\pi V}{2G_4 a} = \frac{\pi}{6} c \ , \tag{104}$$

one gets

$$\mathcal{S}^2 = 2 S_B S_{BH} - S_{BH}^2 \ , \tag{105}$$

or

$$\mathcal{S}^2 + \left(S_{BH} - S_B \right)^2 = S_B^2 \ , \tag{106}$$

which gives the dynamical bound for cosmological entropies. It is quite possible that the origin of cosmological entropy bounds in quantum gravity should be searched in this direction.

8 Summary

In summary, we reviewed the New Brane World [1, 4, 5] and the attempt to supersymmetrize the quantum-induced dilatonic New Brane World [22]. Furthermore it was shown [20] that inside d5 AdS BH the inflationary brane induced by CFT quantum effects in accordance with AdS/CFT may occur in the same way as in refs.[1, 4, 5].

It is shown that for a number of superpotentials one can construct flat SUSY dilatonic brane-world or de Sitter dilatonic brane-world where SUSY is broken by quantum effects. The crucial role in the creation of de Sitter brane universe is in account of quantum effects which produce the effective brane tension. The analysis of graviton perturbations for such brane-worlds shows that gravity trapping on the brane occurs.

The stress tensor of the inflationary brane inside d5 AdS BH is completely defined by dual quantum CFT (and also probably, by brane QG) and it is not chosen by hands as it happens often in the traditional brane-world scenarios. The similirity between CFT entropy at the horizon and FRW equations discovered in refs.[15, 18] is extended for the presence of quantum effects in Brane New World. Such approach may be important for the generalization of cosmological entropy bounds in the case of quantum gravity. From another side, it would be interesting to use such study with the purpose of extension of AdS/CFT correspondence for cosmological (AdS) backgrounds [31].

To conclude, New Brane World represents the embedding into brane physics of old known trace-anomaly driven inflationary scenario (for recent review, see [32]). Its relation with AdS/CFT correspondence, possibility to extend it to presence of dilaton or higher derivative bulk terms [34] or supersymmetrize it, natural connection with cosmological entropy bounds indicates to its important role in the construction of realistic theory of early Universe evolution. Last but not least remark is that it maybe naturally discussed in frames of M-theory.

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A Various expressions of AdS

$D = d + 1$-dimensional Euclidean anti de Sitter space can be embedded in $D + 1$-dimensional flat space, whose metric is given by

$$ds^2_{D+1} = \left(dX^1\right)^2 + \left(dX^2\right)^2 + \cdots + \left(dX^D\right)^2 - \left(dX^0\right)^2 \ . \tag{107}$$

The AdS space is the hypersurface given by

$$\left(X^1\right)^2 + \left(X^2\right)^2 + \cdots + \left(X^D\right)^2 - \left(X^0\right)^2 = -l^2 \ . \tag{108}$$

Here l is a constant, which gives the length scale of the AdS. If we define new coordinates U, V and x^i by

$$V = X^D - X^0 \ , \quad U = X^D + X^0 \ , \quad X^i = U x^i \ (i = 1, 2, \cdots, d(= D - 1) \tag{109}$$

and solve (108) with respect to V as a function of U and x^i, we obtain the following metric

$$ds^2_{\rm AdS} = \frac{l^2}{U^2} dU^2 + U^2 \left\{ \left(dx^1\right)^2 + \left(dx^2\right)^2 + \cdots + \left(dx^1\right)^2 \right\} \ . \tag{110}$$

Further if one changes the variable U by $U = {\rm e}^{\frac{y}{l}}$

$$ds^2_{\rm AdS} = dy^2 + {\rm e}^{\frac{2y}{l}} \left\{ \left(dx^1\right)^2 + \left(dx^2\right)^2 + \cdots + \left(dx^1\right)^2 \right\} \ . \tag{111}$$

On the other hand one can choose the polar coordinates for $\left(X^1, X^2, \cdots, X^D\right)$ and let Y be a radial coordinate

$$Y^2 = \left(X^1\right)^2 + \left(X^2\right)^2 + \cdots + \left(X^D\right)^2 \tag{112}$$

Then by deleting X^0 by using (108), we obtain

$$ds^2_{\rm AdS} = \frac{l^2 dY^2}{Y^2 + l^2} + Y^2 d\Omega_d^2 \ . \tag{113}$$

Here $d\Omega_d^2$ is the metric of the d-dimensional sphere. If we change the variable by $Y = l \sinh \frac{y}{l}$

$$ds^2_{\rm AdS} = dy^2 + l \sinh^2 y d\Omega_d^2 \ . \tag{114}$$

One can also choose a coordinate Z by

$$-Z^2 = \left(X^1\right)^2 + \left(X^2\right)^2 + \cdots + \left(X^{D-1}\right)^2 - \left(X^0\right)^2 \ . \tag{115}$$

Then the hypersurface on the D-dimensional AdS with constant Z is the $D - 1 = d$-dimensional AdS. Let the metric of d-dimensional AdS with unit length parameter is given by dH_d^2. Then the metric of D-dimensional AdS is

$$ds^2_{\rm AdS} = \frac{l^2 dZ^2}{Y^2 - l^2} + Z^2 dH_d^2 \ . \tag{116}$$

Further changing the variable by $Z = l \cosh \frac{y}{l}$ one gets

$$ds^2_{\rm AdS} = dy^2 + l \cosh^2 y dH_d^2 \ . \tag{117}$$

References

[1] S.W. Hawking, T. Hertog and H.S. Reall, *Phys.Rev.* **D62** (2000) 043501, hep-th/0003052.

[2] L. Randall and R. Sundrum, *Phys.Rev.Lett.* **83** (1999)4690, hep-th/9906064.

[3] A.H. Chamblin and H.S. Reall, hep-th/9903225. N. Kaloper, *Phys.Rev.* **D60** (1999) 123506, hep-th/9905210; T. Nihei, *Phys.Lett.* **B465** (1999) 81, hep-th/9905487; H. Kim and H. Kim, hep-th/9909053; C. Csaki, M. Graesser, C. Kolda and J. Terning, hep-ph/9906513; J. Cline, C. Crojean and G. Servant, *Phys.Rev.Lett.* **83** (1999) 4245, hep-ph/9906523; D.J. Chung and K. Freese, *Phys.Rev.* **D61** (2000) 023511, hep-ph/9906542; R. Maartens, D.Wands, B. Bassett and T. Heard, hep-ph/9912464; P. Kanti, I. Kogan, K. Olive and M. Pospelov, *Phys.Lett.* **B468** (1999) 31, hep-ph/9909481; P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, *Phys.Lett.* **B477** (2000) 285, hep-th/9910219; S. Mukohyama, T. Shiromizu and K. Maeda, hep-th/9912287; J. Garriga and M. Sasaki, hep-th/9912118; K. Koyama and J. Soda, hep-th/9912118; J. Kim, B. Kyae and H. Min Lee, hep-th/0004005.

[4] S. Nojiri, S.D. Odintsov and S. Zerbini, *Phys.Rev.* **D62** (2000) 064006, hep-th/0001192.

[5] S. Nojiri and S.D. Odintsov, *Phys.Lett.* **B484** (2000) 119, hep-th/0004097.

[6] T. Ghergetta and A. Pomarol,*Nucl.Phys.* **B586** (2000) 141, hep-ph/0003129;hep-ph/0012378; A. Falkowski, Z. Lalak and S. Pokorski,*Phys.Lett.* **B491** (2000) 172, hep-th/0004093; hep-th/0009167; E. Bergshoeff, R. Kallosh and A. Van Proeyen, *JHEP* **0010** (2000) 033, hep-th/0007044; M.J. Duff, J.T. Liu and K.S. Stelle, hep-th/0007120; M. Cvetic, M.J. Duff, J.T. Liu, H. Lu, C.N. Pope and K.S. Stelle, hep-th/0011167; M.A. Luty and R. Sundrum, hep-th/0012158;

[7] K. Behrndt and M. Cvetic, *Phys.Lett.* **B475** (2000) 253, hep-th/9909058; R. Kallosh and A. Linde, *JHEP* **0002** (2000) 005, hep-th/0001071; M. Zucker, hep-th/0009083; H. Nishino and S. Rajpoot, hep-th/0011066.

[8] Ph. Brax and A.C. Davis, hep-th/0011045.

[9] M. Cvetic, H. Lu, C.N. Pope, *Class.Quant.Grav.* **17** (2000) 4867, hep-th/0001002; hep-th/0002054.

[10] J.M. Maldacena, *Adv.Theor.Math.Phys.* **2** (1998)231; E. Witten, *Adv.Theor.Math.Phys.***2** (1998)253; S. Gubser, I.R. Klebanov and A.M. Polyakov,*Phys.Lett.***B428** (1998)105; O. Aharony, S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Phys.Rept.* **323** (2000) 183, hep-th/9905111.

[11] S. Nojiri, O. Obregon and S.D. Odintsov, *Phys.Rev.* **D62** (2000) 104003, hep-th/0005127; S. Nojiri, O. Obregon, S.D. Odintsov and V.I. Tkach, hep-th/0101003; S. Nojiri, S.D. Odintsov and K.E. Osetrin, hep-th/0009059, *Phys.Rev.* **D**, to appear.

[12] S.W. Hawking, *Comm.Math.Phys.* **43** (1974) 199.

[13] G.'t Hooft, gr-qc/9312026; L. Susskind, *J.Math.Phys.* **36** (1995) 6337; W. Fischler and L. Susskind, hep-th/9806039; R. Easther and D. Lowe, hep-th/9902088, *Phys.Rev.Lett.* **82** (1999) 4967; D. Bak and S. Rey, hep-th/9902173,

Class.Quant.Grav. **17** (2000) L83; N. Kaloper and A. Linde, hep-th/9904120; R. Bousso, hep-th/9905177, *JHEP* **9907** (1999) 004; B. Wang, E. Abdalla and T. Osada, *Phys.Rev.Lett.* **85** (2000) 5507.

[14] S.W. Hawking, J. Maldacena and A. Strominger, hep-th/0002145.

[15] E. Verlinde, hep-th/0008140.

[16] D. Kutasov and F. Larsen,hep-th/0009244; F.-L. Lin, hep-th/0010127; D. Klemm, A. Petcou and G. Siopsis, hep-th/0101076; B. Wang, E. Abdalla and R.-K. Su, hep-th/0101073; Y. S. Myung, hep-th/0102184, hep-th/0103241; R.-G. Cai, hep-th/0102113; R. Brustein, S. Foffa and G. Veneziano, hep-th/0101083; A. Biswas and S. Mukherji, hep-th/0102138; D. Birminghem and S. Mokhtari, hep-th/0103108; D. Klemm, A. Petkou, G. Siopsis, and D. Zanon, hep-th/0104141; N.J. Kim, H.W. Lee and Y.S. Myung, hep-th/0104159; D. Youm, hep-th/0105036, hep-th/0105093; S. Nojiri, O. Obregon, S.D. Odintsov, H. Quevedo and M.P. Ryan, hep-th/0105052; R.-G. Cai,Y.S. Myung and N. Ohta,hep-th/0105070.

[17] S. Nojiri and S.D. Odintsov, hep-th/0011115.

[18] I. Savonije and E. Verlinde, hep-th/0102042.

[19] L. Randall and R. Sundrum, *Phys.Rev.Lett.* **83** (1999) 3370, hep-th/9905221.

[20] S. Nojiri and S.D. Odintsov, hep-th/0102042.

[21] L. Anchordoqui, C. Nunez and K. Olsen, hep-th/0007064; K. Koyama and J. Soda, hep-th/0101164; T. Shiromizu and D. Ida, hep-th/0102035; M. Perez-Victoria, hep-th/0105048.

[22] S. Nojiri and S.D. Odintsov, hep-th/0102032, to appear in *Phys.Rev.* **D**.

[23] S. Nojiri, S.D. Odintsov and S. Ogushi, *Phys.Rev* **D62** (2000) 124002. hep-th/0001122.

[24] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, Effective Action in Quantum Gravity, IOP Publishing, Bristol and Philadelphia 1992.

[25] I. Antoniadis and E. Mottola, *Phys.Rev.* **D45**(1992) 2013; S.D. Odintsov, *Z.Phys.* **C54** (1992) 531.

[26] A. Starobinsky, *Phys.Lett.* **B91** (1980) 99; S.G. Mamaev and V.M. Mostepanenko, *JETP* **51** (1980) 9.

[27] M. Gunaydin, L.J. Romans, N.P. Warner, *Nucl.Phys.* **B272** (1986) 598.

[28] A. Karch and L. Randall, hep-th/0011156.

[29] I.I. Kogan, S. Mouslopoulos and A. Papazoglou, hep-th/0011141.

[30] S. Hawking and D. Page, *Comm.Math.Phys.* **87** (1983) 577.

[31] S. Nojiri and S.D. Odintsov, hep-th/0008160, *Phys.Lett.* **B494** (2000) 135.

[32] S.W. Hawking, T. Hertog and H.S. Reall, hep-th/0010232; K. Hamada, hep-th/0101100.

[33] J.L. Cardy, *Nucl.Phys.* **B270** (1986) 967.

[34] S. Nojiri and S.D. Odintsov, *JHEP* **0007**(2000) 049.